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Correlation Coefficient Measures for Probabilistic Single Valued Neutrosophic Hesitant Fuzzy Sets and Its Application in Supply Chain Management

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Abstract

Here, we introduce the Correlation Coefficient (CC) measures for Probabilistic Single-Valued Neutrosophic Hesitant Fuzzy Sets (PSVNHFSs), aiming to address the complexity of decision-making processes that involve uncertainty, hesitation, and probabilistic elements. The proposed measures offer a systematic approach to calculate the CC between PSVNHFSs by considering the Truth-Membership Hesitancy Degree (TMHD), Indeterminacy-Membership Hesitancy Degree (IMHD), and Falsity-Membership Hesitancy Degree (FMHD). Additionally, the paper introduces a Weighted Correlation Coefficient (WCC) method, allowing for differential weighting based on the risk preferences of Decision-Makers (DMs) and the relative importance of truth, indeterminacy, and falsity degrees. The proposed measure is applied to a Multi-Attribute Decision-Making (MADM) problem in Supply Chain Management (SCM), demonstrating its utility in selecting the best Supplier among multiple Suppliers. The application showcases the impact of attribute weights and risk preferences on supplier rankings, highlighting the measure's flexibility and robustness in real-world scenarios. The results indicate that the proposed CC and WCC measures can significantly enhance decision-making processes in environments characterized by uncertainty and hesitancy.

Keywords: Correlation coefficient, Probabilistic single-valued neutrosophic hesitant fuzzy set, Multi-attribute decision-making, Supply chain management.

1 | Introduction

Over the years, Supply Chain Management (SCM) has emerged as a crucial factor in ensuring the efficient and effective delivery of products and services. Also, SCM encompasses a broad range of activities, including

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procurement, production, distribution, and logistics, all aimed at optimizing the flow of goods, information, and finances from suppliers to end customers. The complexity of these activities often involves a high degree of uncertainty and imprecision, necessitating advanced mathematical and decision-making tools for effective management. To address cognitive uncertainty, indeterminacy and imprecision, Zadeh [1] introduced the concept of Fuzzy Sets (FSs). FSs were later extended for various applications, including Atanassov's Intuitionistic Fuzzy Sets (IFSs) [2] and Type-2 FSs [3]. Mathematically, IFS is equivalent to Interval-Valued Fuzzy Sets (IVFSs) and incorporates membership, non-membership and hesitant functions to describe vagueness and uncertainty. Type-2 FS and its extension to type-n FSs [4] treat the membership function as a FS and making IFS, which includes crisp interval values, i.e. a special case of Type-2 FSs. These extensions aim to comprehensively describe the degree of uncertainty through the general membership degree. In practical scenarios, Decision-Makers (DMs) encounter difficulties like time constraints, complex problems, restricted capacity to process information and a lack of familiarity with public domain resources and relevant information. These challenges hinder their ability to accurately assess decision parameters in Multi-Attribute Decision-Making (MADM) contexts. Consequently, the preference information provided by experts or DMs may be incomplete or imprecise. To address these challenges, Torra [5] and Torra & Narukawa [6] introduced the Hesitant Fuzzy Set (HFS) in which the membership value of each element includes a set of possible values between zero and one. Several studies have focused on entropy, information measures, operational rules, and Multi-Attribute Decision Making (MADM) methods for HFSs [8–12] introduced probability DHFSs and proposed the concepts of Probabilistic Dual Hesitant Fuzzy Sets (PDHFSs) and Probabilistic Dual Hesitant Fuzzy Elements (PDHFEs). They also provided the basic operational rules for these concepts. Additionally, they compared two PDHFEs using the score function and deviation degree. Also, they presented a formula for calculating the entropy of a PDHFE. Since the introduction of PDHFSs, several studies have been conducted on this topic. Thus, the theory of DHFS enables the extension of FSs, IFSs, HFSs, and Fuzzy Multisets (FMS) from a logical perspective. The Neutrosophic Set (NS), first proposed by Smarandache [13], [14] extends the concept of an Intuitionistic Fuzzy Set (IFS) from a philosophical perspective. The terms "neutrosophy" and "neutrosophic" were introduced by Smarandache in his 1998 book. NSs are characterized by separate membership functions for truth, indeterminacy, and falsity. In MADM, DMs can represent their ratings through NSs, which adeptly address indeterminate and inconsistent information. This sets NSs apart from IFSs and FSs, which are limited to managing only incomplete or partial information. Wang et al. [15] initiated the theory of Single-Valued Neutrosophic Set (SVNS) and provided related set theoretical operator definitions. Ye [16] introduced the Single-Valued Neutrosophic Hesitant Fuzzy Set (SVNHFS), integrating SVNSs and HFSs. This new set includes FSs, IFSs, HFSs, FMSs, DHFSs, and SVNSs. Additionally, some properties of Soft-Valued Neutrosophic Hesitant Fuzzy Sets (SVNHFSs) were explored. SVNHFSs are defined by independent membership functions for truth-hesitancy, indeterminacy-hesitancy, and falsity-hesitancy. Some study proposed the Probabilistic Single-Valued Neutrosophic Hesitant Fuzzy Set (PSVNHFS), also known as the Probabilistic Interval Neutrosophic Hesitant Fuzzy Set (PINHFS) based on the concepts of FSs, HFS, PDHFS, NS, and Interval-Valued Neutrosophic Hesitant Fuzzy Sets (IVNHFS) and utilized it to address MADM problems under probabilistic IVNHFS circumstances.

Furthermore, the notion of correlation plays a vital role in managing uncertain information and has been widely utilized in various practical decision-making scenarios, such as pattern recognition, decision analysis, SCM, market forecasting, and machine learning. For example, Malik et al. [18] proposed a new Weighted Correlation Coefficient (WCC) measure for IFSs, ranging between [-1, 1], in which the weights were assigned using the cosine entropy measure. Karaaslan [20] explored the Correlation Coefficient (CC) measures for two NSs, interval-neutrosophic and neutrosophic refined sets and discussed their applications in multi-criteria decision-making problems. Iryna et al. [21] proposed a Interval Single-Valued Neutrosophic Set (ISVNS) algorithm for handling uncertain and inaccurate information in fault diagnosis, utilizing triangular fuzzy numbers and an improved weighted CC to enhance accuracy and meet practical diagnostic needs. Radha et al. [22] introduced a correlation measure for Penta Partitioned Neutrosophic Pythagorean Sets (PNPSs) and Interval-valued PNPS (IVPNPS), discussing its properties and applying it to COVID injection data. The proposed measure extends correlation concepts from Pythagorean FSs and penta-partitioned NSs. Fasihi [23]

reviews Multi-Criteria Analysis (MCA) techniques applied in renewable energy supply chain decision-making. Ahmadabadi et al. [24] present a model for factors impacting Supply Chain Resilience (SCR) using hesitant fuzzy TOPSIS and a meta-synthesis approach, identifying key dimensions like communication, production and crisis management to guide strategic resilience in the tile and ceramic industry. Banihashemi et al.[25] identified key challenges in implementing Green Supply Chain Management (GSCM) in construction, with green design ranked as the most critical component of using the fuzzy Best-Worst Method (BWM). Khanaposhtani [26] introduces a novel MADM method for interval data utilizing SVM techniques, comparing its accuracy against the interval TOPSIS method. Palanikumar et al. [27] utilize Fermatean Vague Normal Sets (FVNS) to address MADM challenges by introducing log-based operators for weighted averaging and geometric calculations. Nafei et al. [28] present a novel decision-making framework that integrates TOPSIS with Neutrosophic Triplets (NTs), enhancing accuracy and computational efficiency in MADM scenarios. Bhat [19] introduces an enhanced AHP group decision-making model utilizing neutrosophic trapezoidal numbers, addressing limitations in the existing model related to the reciprocal property of pairwise comparison matrices. Comparative analysis demonstrates the superiority of this revised model in real-world decision-making scenarios. Hesami [17] introduces a hybrid ANP-TOPSIS framework for strategic supplier selection in reverse logistics, addressing rough uncertainty in the electronics industry. This approach highlights the effectiveness of combining MCDM techniques to handle complex interdependencies and uncertain information in supplier evaluation. Mehmood et al. [12] explore entropy and similarity measures within vague soft sets, focusing on techniques relevant to measurements in relation to crisp points in a given space. Ning et al. [10] proposed a novel CC for PDHFSs, aiming to align with the real-world range of [-1,1] rather than [0,1] and used it in the MADM approach. Şahin and Liu [7] explored the CC of SVNHFSs, examined their properties and established a decision-making method using weighted CCs. Existing correlation measures for various fuzzy set extensions have found applications in areas such as investment analysis, medical diagnosis and risk assessment. However, they are not applicable for managing PSVNHF information. The PSVNHFSs allow DMs to handle uncertainty and hesitation with greater precision, especially in complex datasets where probabilistic elements are crucial. This study aims to fill this gap by providing correlation measures directly supporting decision-making in uncertain and ambiguous environments, as seen in financial forecasting or predictive modeling in healthcare and other sectors. Inspired by the concept of CC developed for HFSs, IFSs, DHFSs, and SVNSs, we introduced:

I. The CC measures for PSVNHFSs aim to address the complexity of decision-making processes that involve uncertainty, hesitation, and probabilistic elements. The proposed measures provide a systematic approach to calculating the CC between PSVNHFSs by considering TMHD, IMHD and FMHD. Additionally

II. The WCC measure allows for differential weighting based on DMs' risk preferences and the relative importance of truth, indeterminacy and falsity degrees.

This advanced methodological framework is applied to a MADM problem in SCM, demonstrating its utility in selecting the best Supplier among multiple candidates. The application highlights the impact of attribute weights and risk preferences on supplier rankings, showcasing the method's flexibility and robustness in real-world situations. The results indicate that the proposed CC and WCC measures can significantly enhance decision-making processes in environments characterized by uncertainty and hesitancy, providing a powerful tool for SCM and other complex decision-making domains.

The rest of the paper is organized as follows. Section 2 presents fundamental definitions and reviews related work. Section 3 introduces the proposed CC and weighted CC measures, along with proofs of their validity. Section 4 outlines the decision-making Algorithm based on the WCC for the PSVNHFS. Section 5 comprehensively analyzes how various weights and risk preferences influence supplier selection. Lastly, Section 6 concludes the paper with a summary of the findings and suggestions for future research directions.

2 | Preliminaries

Definition 1. A NS on a fixed set X is defined as

$$\mathfrak{T}_N = \{(\ell, t_N(\ell), i_N(\ell), f_N(\ell)) | \ell \in X\},$$

where $t_N(\ell), i_N(\ell)$ and $f_N(\ell)$ are the truth-membership degree, indeterminacy-membership degree and falsity-membership degree to the set X of ℓ, respectively. $t_N: X \to]0^-, 1^+[, i_N: X \to]0^-, 1^+[, f_N: X \to]0^-, 1^+[$ with $0^- \le \sup t_N(\ell) + \sup i_N(\ell) + \sup f_N(\ell) \le 3^+$ for all $\ell \in X$.

Definition 2. A SVNS on a fixed set X is defined as

$$\mathfrak{T}_N = \{(\ell, t_N(\ell), i_N(\ell), f_N(\ell)) | \ell \in X\},$$

where $t_N(\ell), i_N(\ell)$ and $f_N(\ell)$ are the truth-membership degree, indeterminacy-membership degree and falsity-membership degree to the set X of ℓ, respectively. $t_N: X \to [0,1], i_N: X \to [0,1], f_N: X \to [0,1]$ with $0 \le t_N(\ell) + i_N(\ell) + f_N(\ell) \le 3$ for all $\ell \in X$.

Definition 3. A HFS $\mathfrak{T}$ on a fixed set X is defined as

$$\mathfrak{T} = \{(\ell_i, h(\ell_i)) | \ell_i \in X\},$$

where $h(\ell_i) \in [0,1]$ is the hesitancy membership degree of all $\ell_i \in X$.

Definition 4. A DHFS $\mathfrak{T}$ on a fixed set X is defined as

$$\mathfrak{T} = \{(\ell_i, h(\ell_i), g(\ell_i)) | \ell_i \in X\},$$

where $h(\ell_i)$ and $g(\ell_i) \in [0,1]$ are the hesitancy membership and non-membership degrees, respectively, of all $\ell_i \in X$ with $0 \le \alpha, \gamma \le 1$ and $0 \le \alpha^+ + \gamma^+ \le 1$ where $\alpha \in h(\ell_i), \beta \in g(\ell_i), \alpha^+ \in h^+(\ell_i)$ and $\beta^+ \in g^+(\ell_i)$.

Definition 5. A SVNHS $\mathfrak{T}$ on a fixed set X is defined as

$$\mathfrak{T}_N = \{(\ell, t_N(\ell), i_N(\ell), f_N(\ell)) | \ell \in X\},$$

where $t_N(\ell), i_N(\ell)$ and $f_N(\ell)$ are the TMHD, IMHD and FMHD to the set X of ℓ, respectively. $0 \le \vartheta, \mu, \pi \le 1$; $0 \le \vartheta^+, \mu^+, \pi^+ \le 1$; $\vartheta \in t_N(\ell), \mu \in i_N$ and $\pi \in f_N(\ell)$;$\vartheta^+ \in t_N^+(\ell) = \cup_{\vartheta \in t_N(\ell)} \max \nu$, $\mu^+ \in i_N^+(\ell) = \cup_{\mu \in i_N} \max \mu$, $\pi^+ \in f_N^+(\ell) = \cup_{\pi \in f_N(\ell)} \max \pi$.

Definition 6. A PSVNHFS on a fixed set X is defined as

$$\mathfrak{T}_N = \{\langle \ell, t_N(\ell)|p_I(\ell), i_N(\ell)|p_{II}(\ell), f_N(\ell)|p_{III}(\ell)\rangle | \ell \in X,$$

where $t_N(\ell), i_N(\ell)$ and $f_N(\ell)$ are the TMHD, IMHD and FMHD to the set X of ℓ, respectively. $p_I(\ell), p_{II}(\ell)$ and $p_{III}(\ell)$ are the corresponding probabilistic information for the three types of degree and $0 \le \vartheta, \mu, \pi \le 1$; $0 \le \vartheta^+, \mu^+, \pi^+ \le 1$; $p_I(\ell), p_{II}(\ell), p_{III}(\ell) \in [0,1]$; $\sum_{!=1}^{\#t_N} p_{I!}(\ell) = \sum_{!=1}^{\#i_N} p_{II!}(\ell) = \sum_{!=1}^{\#f_N} p_{III!}(\ell) = 1$; $\vartheta \in t_N(\ell), \mu \in i_N$ and $\pi \in f_N(\ell)$;$\vartheta^+ \in t_N^+(\ell) = \cup_{\vartheta \in t_N(\ell)} \max \nu$, $\mu^+ \in i_N^+(\ell) = \cup_{\mu \in i_N} \max \mu$, $\pi^+ \in f_N^+(\ell) = \cup_{\pi \in f_N(\ell)} \max \pi$, $p_{I!}(\ell) \in p_I(\ell), p_{II!}(\ell) \in p_{II}(\ell), p_{III!}(\ell) \in p_{III}(\ell)$. The symbols $\#t_N$, $\#i_N$ and $\#f_N$ are the total observations in $t_N(\ell)|p_I(\ell)$, $i_N(\ell)|p_{II}(\ell)$ and $f_N(\ell)|p_{III}(\ell)$ respectively.

Example 1. Suppose we have a set of three employees, X = {e1, e2, e3} and we want to evaluate their performance using a PSVNHFS.

The PSVNHFS A can be defined as A = {(e1, {0.8|0.6,0.7|0.4)}, {0.2|0.7, 0.3|0.3}, {0.1|0.8,0.2|0.2)}), (e2, {0.9|0.5,0.8|0.5}, {0.1|0.6, 0.2| 0.4}, {0.1|0.7,0.2|0.3}), (e3, {0.7|0.4,0.6|0.6}, {0.3|0.5,0.4|0.5}, {0.2|0.6,0.3|0.4})}.

3 | Correlation Coefficient for Probabilistic Single Valued Neutrosophic Hesitant Fuzzy Sets

3.1 | Proposed Correlation Coefficient between Two PSVNHFSs

Let us consider $\mathcal{X} = \langle t_{\mathcal{X}}|\wp_{\mathrm{I}\mathcal{X}}, i_{\mathcal{X}}|\wp_{\mathrm{II}\mathcal{X}}, f_{\mathcal{X}}|\wp_{\mathrm{III}\mathcal{X}}\rangle$ and $\mathcal{Y} = \langle t_{\mathcal{Y}}|\wp_{\mathrm{I}\mathcal{Y}}, i_{\mathcal{Y}}|\wp_{\mathrm{II}\mathcal{Y}}, f_{\mathcal{Y}}|\wp_{\mathrm{III}\mathcal{Y}}\rangle$ be two PSVNHFS on X. In some situations, the number of elements in set $t_{\mathcal{X}}$ and $t_{\mathcal{Y}}$ may be different. Let $\mathrm{I}_{\ell_i} = \max\left(I(t_{\mathcal{X}}), I(t_{\mathcal{Y}})\right)$, $\Lambda_{\ell_i} = \max\left(\mathrm{I}(i_{\mathcal{X}}), \mathrm{I}(i_{\mathcal{Y}})\right)$ and $\Upsilon_{\ell_i} = \max\left(\mathrm{I}(f_{\mathcal{X}}), \mathrm{I}(f_{\mathcal{Y}})\right)$ for each $\ell_i \in \mathrm{X}$ and $\mathrm{I}(t_{\mathcal{X}}), \mathrm{I}(t_{\mathcal{Y}}), \mathrm{I}(i_{\mathcal{X}}), \mathrm{I}(i_{\mathcal{Y}}), \mathrm{I}(f_{\mathcal{X}})$ and $\mathrm{I}(f_{\mathcal{Y}})$ are the number of elements in sets $t_{\mathcal{X}}, t_{\mathcal{Y}}, i_{\mathcal{X}}, i_{\mathcal{Y}}, f_{\mathcal{X}}$ and $f_{\mathcal{Y}}$ respectively. Here, we will use the following criterion to make the elements of TMHD, IMHD and FMHD equal:

I. If $\mathrm{I}(t_{\mathcal{X}}) > \mathrm{I}(t_{\mathcal{Y}})$, firstly, we rank the TMHD, IMHD, and FMHD of each PSVNHFS in descending order. Then, we add the element with the smallest value in $t_{\mathcal{X}}$ to $t_{\mathcal{Y}}$, with the probability of selecting this element being zero.

II. If $\mathrm{I}(i_{\mathcal{X}}) > \mathrm{I}(i_{\mathcal{Y}})$, firstly, we rank the TMHD, IMHD, and FMHD of each PSVNHFS in descending order. Then, we add the element with the smallest value in $i_{\mathcal{X}}$ to $i_{\mathcal{Y}}$, with the probability of selecting this element being zero.

III. If $\mathrm{I}(f_{\mathcal{X}}) > \mathrm{I}(f_{\mathcal{Y}})$, firstly, we rank the TMHD, IMHD, and FMHD of each PSVNHFS in descending order. Then, we add the element with the smallest value in $f_{\mathcal{X}}$ to $f_{\mathcal{Y}}$, with the probability of selecting this element being zero.

If $\mathcal{X} = \langle t_{\mathcal{X}}|\wp_{\mathrm{I}\mathcal{X}}, i_{\mathcal{X}}|\wp_{\mathrm{II}\mathcal{X}}, f_{\mathcal{X}}|\wp_{\mathrm{III}\mathcal{X}}\rangle$ and $\mathcal{Y} = \langle t_{\mathcal{Y}}|\wp_{\mathrm{I}\mathcal{Y}}, i_{\mathcal{Y}}|\wp_{\mathrm{II}\mathcal{Y}}, f_{\mathcal{Y}}|\wp_{\mathrm{III}\mathcal{Y}}\rangle$ be two PSVNHFS defined on $\mathrm{X} = (\ell_1, \ell_2, \ldots, \ell_n)$, then the CC between $\mathcal{X}$ and $\mathcal{Y}$ is defined as

$$\mathfrak{p}(\mathcal{X}, \mathcal{Y}) = \frac{1}{3}\left(\mathfrak{p}_t(\mathcal{X}, \mathcal{Y}) + \mathfrak{p}_i(\mathcal{X}, \mathcal{Y}) + \mathfrak{p}_f(\mathcal{X}, \mathcal{Y})\right), \tag{1}$$

where

$$\mathfrak{p}_t(\mathcal{X}, \mathcal{Y}) = \frac{\sum_{i=1}^{n}\left[\left(\sum_{j=1}^{\mathrm{I}_{\ell_i}} t_{\mathcal{X}_j}(\ell_i)\wp_{\mathrm{I}\mathcal{X}_j}(\ell_i) - \overline{t_{\mathcal{X}}}\right)\left(\sum_{j=1}^{\mathrm{I}_{\ell_i}} t_{\mathcal{Y}_j}(\ell_i)\wp_{\mathrm{I}\mathcal{Y}_j}(\ell_i) - \overline{t_{\mathcal{Y}}}\right)\right]}{\sqrt{\sum_{i=1}^{n}\left[\left(\sum_{j=1}^{\mathrm{I}_{\ell_i}} t_{\mathcal{X}_j}(\ell_i)\wp_{\mathrm{I}\mathcal{X}_j}(\ell_i) - \overline{t_{\mathcal{X}}}\right)^2\right]}\sqrt{\sum_{i=1}^{n}\left[\left(\sum_{j=1}^{\mathrm{I}_{\ell_i}} t_{\mathcal{Y}_j}(\ell_i)\wp_{\mathrm{I}\mathcal{Y}_j}(\ell_i) - \overline{t_{\mathcal{Y}}}\right)^2\right]}}. \tag{2}$$

$$\mathfrak{p}_i(\mathcal{X}, \mathcal{Y}) = \frac{\sum_{i=1}^{n}\left[\left(\sum_{j=1}^{\Lambda_{\ell_i}} i_{\mathcal{X}_j}(\ell_i)\wp_{\mathrm{II}\mathcal{X}_j}(\ell_i) - \overline{i_{\mathcal{X}}}\right)\left(\sum_{j=1}^{\Lambda_{\ell_i}} i_{\mathcal{Y}_j}(\ell_i)\wp_{\mathrm{II}\mathcal{Y}_j}(\ell_i) - \overline{i_{\mathcal{Y}}}\right)\right]}{\sqrt{\sum_{i=1}^{n}\left[\left(\sum_{j=1}^{\Lambda_{\ell_i}} i_{\mathcal{X}_j}(\ell_i)\wp_{\mathrm{II}\mathcal{X}_j}(\ell_i) - \overline{i_{\mathcal{X}}}\right)^2\right]}\sqrt{\sum_{i=1}^{n}\left[\left(\sum_{j=1}^{\Lambda_{\ell_i}} i_{\mathcal{Y}_j}(\ell_i)\wp_{\mathrm{II}\mathcal{Y}_j}(\ell_i) - \overline{i_{\mathcal{Y}}}\right)^2\right]}}. \tag{3}$$

$$\mathfrak{p}_f(\mathcal{X}, \mathcal{Y}) = \frac{\sum_{i=1}^{n}\left[\left(\sum_{j=1}^{\Upsilon_{\ell_i}} f_{\mathcal{X}_j}(\ell_i)\wp_{\mathrm{III}\mathcal{X}_j}(\ell_i) - \overline{f_{\mathcal{X}}}\right)\left(\sum_{j=1}^{\Upsilon_{\ell_i}} f_{\mathcal{Y}_j}(\ell_i)\wp_{\mathrm{III}\mathcal{Y}_j}(\ell_i) - \overline{f_{\mathcal{Y}}}\right)\right]}{\sqrt{\sum_{i=1}^{n}\left[\left(\sum_{j=1}^{\Upsilon_{\ell_i}} f_{\mathcal{X}_j}(\ell_i)\wp_{\mathrm{III}\mathcal{X}_j}(\ell_i) - \overline{f_{\mathcal{X}}}\right)^2\right]}\sqrt{\sum_{i=1}^{n}\left[\left(\sum_{j=1}^{\Upsilon_{\ell_i}} f_{\mathcal{Y}_j}(\ell_i)\wp_{\mathrm{III}\mathcal{Y}_j}(\ell_i) - \overline{f_{\mathcal{Y}}}\right)^2\right]}}. \tag{4}$$

And the mean of the TMHD of PSVNHFS $\mathcal{X}$ and $\mathcal{Y}$ are

$$\overline{t_{\mathcal{X}}} = \frac{1}{n}\sum_{i=1}^{n}\left(\sum_{j=1}^{\mathrm{I}_{\ell_i}} t_{\mathcal{X}_j}(\ell_i)\,\wp_{\mathrm{I}\mathcal{X}_j}(\ell_i)\right). \tag{5}$$

$$\overline{t_{\mathcal{Y}}} = \frac{1}{n}\sum_{i=1}^{n}\left(\sum_{j=1}^{\mathrm{I}_{\ell_i}} t_{\mathcal{Y}_j}(\ell_i)\,\wp_{\mathrm{I}\mathcal{Y}_j}(\ell_i)\right). \tag{6}$$

The mean of the IMHD of PSVNHFS $\mathcal{X}$ and $\mathcal{Y}$ are

$$\overline{\imath_{\mathcal{X}}} = \frac{1}{n}\sum_{i=1}^{n}\left(\sum_{j=1}^{\Lambda_{\ell_i}} \imath_{\mathcal{X}_j}(\ell_i)\, \wp_{II\mathcal{X}_j}(\ell_i)\right). \tag{7}$$

$$\overline{\imath_{\mathcal{Y}}} = \frac{1}{n}\sum_{i=1}^{n}\left(\sum_{j=1}^{\Lambda_{\ell_i}} \imath_{\mathcal{Y}_j}(\ell_i)\, \wp_{II\mathcal{Y}_j}(\ell_i)\right). \tag{8}$$

The mean of the FMHD of PSVNHFS $\mathcal{X}$ and $\mathcal{Y}$ are

$$\overline{\mathcal{f}_{\mathcal{X}}} = \frac{1}{n}\sum_{i=1}^{n}\left(\sum_{j=1}^{\Upsilon_{\ell_i}} \mathcal{f}_{\mathcal{X}_j}(\ell_i)\, \wp_{III\mathcal{X}_j}(\ell_i)\right). \tag{9}$$

$$\overline{\mathcal{f}_{\mathcal{Y}}} = \frac{1}{n}\sum_{i=1}^{n}\left(\sum_{j=1}^{\Upsilon_{\ell_i}} \mathcal{f}_{\mathcal{Y}_j}(\ell_i)\, \wp_{III\mathcal{Y}_j}(\ell_i)\right). \tag{10}$$

Theorem 1. Let $\mathcal{X} = \langle \mathcal{t}_{\mathcal{X}}|\wp_{I\mathcal{X}}, \imath_{\mathcal{X}}|\wp_{II\mathcal{X}}, \mathcal{f}_{\mathcal{X}}|\wp_{III\mathcal{X}}\rangle$ and $\mathcal{Y} = \langle \mathcal{t}_{\mathcal{Y}}|\wp_{I\mathcal{Y}}, \imath_{\mathcal{Y}}|\wp_{II\mathcal{Y}}, \mathcal{f}_{\mathcal{Y}}|\wp_{III\mathcal{Y}}\rangle$ be two PSVNHFS defined on $X = (\ell_1, \ell_2, \ldots, \ell_n)$, then the CC between $\mathcal{X}$ and $\mathcal{Y}$ will satisfy the following conditions:

$\mathfrak{p}(\mathcal{X},\mathcal{Y}) = \mathfrak{p}(\mathcal{Y},\mathcal{X})$,

$\mathfrak{p}(\mathcal{X},\mathcal{Y}) = 1$ if $\mathcal{X} = \mathcal{Y}$,

$-1 \leq \mathfrak{p}(\mathcal{X},\mathcal{Y}) \leq 1$.

Proof: it can easily be proved by interchanging $\mathcal{X}$ to $\mathcal{Y}$ in $\mathfrak{p}_{\mathcal{t}}(\mathcal{X},\mathcal{Y}), \mathfrak{p}_{\imath}(\mathcal{X},\mathcal{Y})$ and $\mathfrak{p}_{\mathcal{f}}(\mathcal{X},\mathcal{Y})$ and substituting the obtained values in values in $\mathfrak{p}(\mathcal{X},\mathcal{Y})$. We will get

$$\mathfrak{p}(\mathcal{Y},\mathcal{X}) = \frac{1}{3}\left(\mathfrak{p}_{\mathcal{t}}(\mathcal{Y},\mathcal{X}) + \mathfrak{p}_{\imath}(\mathcal{Y},\mathcal{X}) + \mathfrak{p}_{\mathcal{f}}(\mathcal{Y},\mathcal{X})\right) = \mathfrak{p}(\mathcal{X},\mathcal{Y}).$$

Proof: if $\mathcal{X} = \mathcal{Y}$, then from *Eq. (2)*

$$\mathfrak{p}_{\mathcal{t}}(\mathcal{X},\mathcal{X}) = \frac{\sum_{i=1}^{n}\left[\left(\sum_{j=1}^{I_{\ell_i}} \mathcal{t}_{\mathcal{X}_j}(\ell_i)\wp_{I\mathcal{X}_j}(\ell_i) - \overline{\mathcal{t}_{\mathcal{X}}}\right)\left(\sum_{j=1}^{I_{\ell_i}} \mathcal{t}_{\mathcal{X}_j}(\ell_i)\wp_{I\mathcal{X}_j}(\ell_i) - \overline{\mathcal{t}_{\mathcal{X}}}\right)\right]}{\sqrt{\sum_{i=1}^{n}\left[\left(\sum_{j=1}^{I_{\ell_i}} \mathcal{t}_{\mathcal{X}_j}(\ell_i)\wp_{I\mathcal{X}_j}(\ell_i) - \overline{\mathcal{t}_{\mathcal{X}}}\right)^2\right]}\sqrt{\sum_{i=1}^{n}\left[\left(\sum_{j=1}^{I_{\ell_i}} \mathcal{t}_{\mathcal{X}_j}(\ell_i)\wp_{I\mathcal{X}_j}(\ell_i) - \overline{\mathcal{t}_{\mathcal{X}}}\right)^2\right]}},$$

$$\mathfrak{p}_{\mathcal{t}}(\mathcal{X},\mathcal{X}) = \frac{\sum_{i=1}^{n}\left[\left(\sum_{j=1}^{I_{\ell_i}} \mathcal{t}_{\mathcal{X}_j}(\ell_i)\wp_{I\mathcal{X}_j}(\ell_i) - \overline{\mathcal{t}_{\mathcal{X}}}\right)^2\right]}{\sum_{i=1}^{n}\left[\left(\sum_{j=1}^{I_{\ell_i}} \mathcal{t}_{\mathcal{X}_j}(\ell_i)\wp_{I\mathcal{X}_j}(\ell_i) - \overline{\mathcal{t}_{\mathcal{X}}}\right)^2\right]} = 1. \tag{11}$$

Similarly, $\mathfrak{p}_{\imath}(\mathcal{X},\mathcal{X}) = \mathfrak{p}_{\mathcal{f}}(\mathcal{X},\mathcal{X}) = 1$.

By putting these values in *Eq. (1)*, we have

$$\mathfrak{p}(\mathcal{X},\mathcal{X}) = \frac{1}{3}\left(\mathfrak{p}_{\mathcal{t}}(\mathcal{X},\mathcal{X}) + \mathfrak{p}_{\imath}(\mathcal{X},\mathcal{X}) + \mathfrak{p}_{\mathcal{f}}(\mathcal{X},\mathcal{X})\right) = 1. \tag{12}$$

Similarly, we can prove $\mathfrak{p}(\mathcal{Y},\mathcal{Y}) = 1$.

Proof: from Cauchy Schwarz inequality, we know that

$(u_1 v_1 + u_2 v_2 + \cdots + u_n v_n)^2 \leq (u_1^2 + u_2^2 + \cdots + u_n^2)(v_1^2 + v_2^2 + \cdots + v_n^2)$ for all $u_i, v_i \in \mathfrak{R}$.

Using the inequality in *Eq. (2)*, we have

$$\left\{\sum_{i=1}^{n}\left[\left(\sum_{j=1}^{I_{\ell_i}} \mathcal{t}_{\mathcal{X}_j}(\ell_i)\,\wp_{I\mathcal{X}_j}(\ell_i) - \overline{\mathcal{t}_{\mathcal{X}}}\right)\left(\sum_{j=1}^{I_{\ell_i}} \mathcal{t}_{\mathcal{Y}_j}(\ell_i)\,\wp_{I\mathcal{Y}_j}(\ell_i) - \overline{\mathcal{t}_{\mathcal{Y}}}\right)\right]\right\}^2 \leq \sum_{i=1}^{n}\left[\left(\sum_{j=1}^{I_{\ell_i}} \mathcal{t}_{\mathcal{X}_j}(\ell_i)\,\wp_{I\mathcal{X}_j}(\ell_i) - \overline{\mathcal{t}_{\mathcal{X}}}\right)^2\right]\sum_{i=1}^{n}\left[\left(\sum_{j=1}^{I_{\ell_i}} \mathcal{t}_{\mathcal{Y}_j}(\ell_i)\,\wp_{I\mathcal{Y}_j}(\ell_i) - \overline{\mathcal{t}_{\mathcal{Y}}}\right)^2\right]. \tag{13}$$

On taking the square root on both sides of *Eq. (13)*, we have

$$\left|\sum_{i=1}^{n}\left[\left(\sum_{j=1}^{I_{\ell_i}} t_{\mathcal{X}_j}(\ell_i)\,\wp_{I\mathcal{X}_j}(\ell_i)-\overline{t_{\mathcal{X}}}\right)\left(\sum_{j=1}^{I_{\ell_i}} t_{\mathcal{Y}_j}(\ell_i)\,\wp_{I\mathcal{Y}_j}(\ell_i)-\overline{t_{\mathcal{Y}}}\right)\right]\right| \le$$
$$\sqrt{\sum_{i=1}^{n}\left[\left(\sum_{j=1}^{I_{\ell_i}} t_{\mathcal{X}_j}(\ell_i)\,\wp_{I\mathcal{X}_j}(\ell_i)-\overline{t_{\mathcal{X}}}\right)^2\right]}\sqrt{\sum_{i=1}^{n}\left[\left(\sum_{j=1}^{I_{\ell_i}} t_{\mathcal{X}_j}(\ell_i)\,\wp_{I\mathcal{X}_j}(\ell_i)-\overline{t_{\mathcal{X}}}\right)^2\right]}.$$

Which implies

$$\frac{\left|\sum_{i=1}^{n}\left[\left(\sum_{j=1}^{I_{\ell_i}} t_{\mathcal{X}_j}(\ell_i)\wp_{I\mathcal{X}_j}(\ell_i)-\overline{t_{\mathcal{X}}}\right)\left(\sum_{j=1}^{I_{\ell_i}} t_{\mathcal{Y}_j}(\ell_i)\wp_{I\mathcal{Y}_j}(\ell_i)-\overline{t_{\mathcal{Y}}}\right)\right]\right|}{\sqrt{\sum_{i=1}^{n}\left[\left(\sum_{j=1}^{I_{\ell_i}} t_{\mathcal{X}_j}(\ell_i)\wp_{I\mathcal{X}_j}(\ell_i)-\overline{t_{\mathcal{X}}}\right)^2\right]}\sqrt{\sum_{i=1}^{n}\left[\left(\sum_{j=1}^{I_{\ell_i}} t_{\mathcal{Y}_j}(\ell_i)\wp_{I\mathcal{Y}_j}(\ell_i)-\overline{t_{\mathcal{Y}}}\right)^2\right]}} \le 1.$$

Hence, $|\mathfrak{p}_t(\mathcal{X},\mathcal{Y})| \le 1$. Similarly, we can show $|\mathfrak{p}_i(\mathcal{X},\mathcal{Y}) \le 1|$ and $|\mathfrak{p}_f(\mathcal{X},\mathcal{Y}) \le 1|$.

Further, $|\mathfrak{p}(\mathcal{X},\mathcal{Y})| = \frac{1}{3}\left(|\mathfrak{p}_t(\mathcal{X},\mathcal{Y})| + |\mathfrak{p}_i(\mathcal{X},\mathcal{Y})| + |\mathfrak{p}_f(\mathcal{X},\mathcal{Y})|\right)|\mathfrak{p}(\mathcal{X},\mathcal{Y})| \le \frac{1}{3}+\frac{1}{3}+\frac{1}{3}$

$|\mathfrak{p}(\mathcal{X},\mathcal{Y})| \le 1 \rightarrow -1 \le \mathfrak{p}(\mathcal{X},\mathcal{Y}) \le 1$. Hence proved.

Example 2. Let

$$\mathcal{X} = \left\{\begin{matrix} \langle\{0.6|0.2,0.5|0.4,0.3|0.4\},\{0.5|0.4,0.6|0.3,0.3|0.3\},\{0.5|0.2,0.4|0.4,0.2|0.4\}\rangle \\ \langle\{0.3|0.4,0.6|0.3,0.9|0.2,0.2|0.1\},\{0.4|0.3,0.5|0.2,0.3|0.3,0.4|0.2\},\{0.3|0.4,0.5|0.2,0.2|0.3,0.1|0.1\}\rangle \\ \langle\{0.4|0.6,0.3|0.4\},\{0.5|0.7,0.3|0.3\},\{0.9|0.5,0.8|0.5\}\rangle \\ \langle\{0.4|0.3,0.6|0.2,0.5|0.3,0.3|0.2\},\{0.8|0.4,0.6|0.3,0.9|0.2,0.5|0.1\},\{0.5|0.2,0.4|0.4,0.2|0.3,0.1|0.1\}\rangle \\ \langle\{0.4|0.1,0.6|0.4,0.7|0.5\},\{0.5|0.3,0.4|0.4,0.6|0.3\},\{0.9|0.5,0.7|0.2,0.5|0.3\}\rangle \end{matrix}\right\},$$

And

$$\mathcal{Y} = \left\{\begin{matrix} \langle\{0.5|0.1,0.4|0.5,0.2|0.4\},\{0.6|0.3,0.7|0.4,0.4|0.3\},\{0.7|0.4,0.6|0.3,0.4|0.3\}\rangle \\ \langle\{0.5|0.3,0.8|0.2,0.7|0.2,0.3|0.3\},\{0.5|0.2,0.6|0.3,0.4|0.3,0.7|0.2\},\{0.4|0.3,0.6|0.4,0.3|0.2,0.2|0.1\}\rangle \\ \langle\{0.8|0.4,0.5|0.6\},\{0.6|0.2,0.5|0.8\},\{0.5|0.2,0.8|0.8\}\rangle \\ \langle\{0.5|0.2,0.7|0.3,0.6|0.4,0.4|0.1\},\{0.7|0.3,0.5|0.4,0.8|0.1,0.4|0.2\},\{0.6|0.4,0.5|0.3,0.3|0.2,0.4|0.1\}\rangle \\ \langle\{0.5|0.2,0.7|0.4,0.8|0.4\},\{0.6|0.2,0.3|0.4,0.5|0.4\},\{0.8|0.5,0.6|0.2,0.4|0.3\}\rangle \end{matrix}\right\}.$$

Using *Eqs. (5)-(10)*, we can calculate the mean of the TMHD of PSVNHFS $\mathcal{X}$ and $\mathcal{Y}$, the mean of the IMHD of PSVNHFS $\mathcal{X}$ and $\mathcal{Y}$ and the mean of the FMHD of PSVNHFS $\mathcal{X}$ and $\mathcal{Y}$ as follows:

$$\overline{t_{\mathcal{X}}} = \frac{1}{5}\begin{bmatrix} 0.6*0.2+0.5*0.4+0.3*0.4+0.3*0.4+0.6*0.3 \\ +0.9*0.2+0.2*0.1+0.4*0.6+0.3*0.4 \\ +0.4*0.3+0.6*0.2+0.5*0.3+0.3*0.2 \\ +0.4*0.1+0.6*0.4+0.7*0.5 \end{bmatrix} = 0.476.$$

$$\overline{t_{\mathcal{Y}}} = \frac{1}{5}\begin{bmatrix} 0.5*0.1+0.4*0.5+0.2*0.4+0.5*0.3+0.8*0.2 \\ +0.7*0.2+0.3*0.3+0.8*0.4+0.5*0.6 \\ +0.5*0.2+0.7*0.3+0.6*0.4+0.4*0.1 \\ +0.5*0.2+0.7*0.4+0.8*0.4 \end{bmatrix} = 0.556.$$

$$\overline{i_{\mathcal{X}}} = \frac{1}{5}\begin{bmatrix} 0.5*0.4+0.6*0.3+0.3*0.3+0.4*0.3+0.5*0.2 \\ +0.3*0.3+0.4*0.2+0.5*0.7+0.3*0.3+0.8*0.4 \\ +0.6*0.3+0.9*0.2+0.5*0.1+ \\ 0.5*0.3+0.4*0.4+0.6*0.3 \end{bmatrix} = 0.504.$$

$$\overline{i_{\mathcal{Y}}} = \frac{1}{5}\begin{bmatrix} 0.6*0.3+0.7*0.4+0.4*0.3+0.5*0.2+0.6*0.3 \\ +0.4*0.3+0.7*0.2+0.6*0.2+0.5*0.8+0.7*0.3 \\ +0.5*0.4+0.8*0.1+0.4*0.2+ \\ 0.6*0.2+0.3*0.4+0.5*0.4 \end{bmatrix} = 0.53.$$

$$\overline{f_{\mathcal{X}}} = \frac{1}{5}\begin{bmatrix} 0.5*0.2+0.4*0.4+0.2*0.4+0.3*0.4+0.8*0.2 \\ +0.2*0.3+0.1*0.1+0.9*0.5+0.8*0.5 \\ +0.5*0.2+0.4*0.4+0.2*0.3+0.1*0.1 \\ +0.9*0.5+0.7*0.2+0.5*0.3 \end{bmatrix} = 0.522.$$

$$\overline{f_{\mathcal{Y}}} = \frac{1}{5}\begin{bmatrix} 0.7*0.4+0.6*0.3+0.4*0.3+0.4*0.3+0.6*0.4 \\ +0.3*0.2+0.2*0.1+0.5*0.2+0.8*0.8 \\ +0.6*0.4+0.5*0.3+0.3*0.2+0.4*0.1 \\ +0.8*0.5+0.6*0.2+0.4*0.3 \end{bmatrix} = 0.578.$$

Using *Eqs. (2)-(4)*, we have

$$\mathfrak{p}_t(\mathcal{X},\mathcal{Y}) = \frac{\sum_{i=1}^{n}\left[\left(\sum_{j=1}^{I_{\ell_i}} t_{\mathcal{X}_j}(\ell_i)\wp_{I\mathcal{X}_j}(\ell_i)-\overline{t_{\mathcal{X}}}\right)\left(\sum_{j=1}^{I_{\ell_i}} t_{\mathcal{Y}_j}(\ell_i)\wp_{I\mathcal{Y}_j}(\ell_i)-\overline{t_{\mathcal{Y}}}\right)\right]}{\sqrt{\sum_{i=1}^{n}\left[\left(\sum_{j=1}^{I_{\ell_i}} t_{\mathcal{X}_j}(\ell_i)\wp_{I\mathcal{X}_j}(\ell_i)-\overline{t_{\mathcal{X}}}\right)^2\right]}\sqrt{\sum_{i=1}^{n}\left[\left(\sum_{j=1}^{I_{\ell_i}} t_{\mathcal{Y}_j}(\ell_i)\wp_{I\mathcal{Y}_j}(\ell_i)-\overline{t_{\mathcal{Y}}}\right)^2\right]}} = \frac{0.02162}{0.0554} = 0.3901.$$

$$\mathfrak{p}_i(\mathcal{X},\mathcal{Y}) = \frac{\sum_{i=1}^{n}\left[\left(\sum_{j=1}^{\Lambda_{\ell_i}} i_{\mathcal{X}_j}(\ell_i)\wp_{II\mathcal{X}_j}(\ell_i)-\overline{i_{\mathcal{X}}}\right)\left(\sum_{j=1}^{\Lambda_{\ell_i}} i_{\mathcal{Y}_j}(\ell_i)\wp_{II\mathcal{Y}_j}(\ell_i)-\overline{i_{\mathcal{Y}}}\right)\right]}{\sqrt{\sum_{i=1}^{n}\left[\left(\sum_{j=1}^{\Lambda_{\ell_i}} i_{\mathcal{X}_j}(\ell_i)\wp_{II\mathcal{X}_j}(\ell_i)-\overline{i_{\mathcal{X}}}\right)^2\right]}\sqrt{\sum_{i=1}^{n}\left[\left(\sum_{j=1}^{\Lambda_{\ell_i}} i_{\mathcal{Y}_j}(\ell_i)\wp_{II\mathcal{Y}_j}(\ell_i)-\overline{i_{\mathcal{Y}}}\right)^2\right]}} = \frac{0.0081}{0.029361} = 0.2759.$$

$$\mathfrak{p}_f(\mathcal{X},\mathcal{Y}) = \frac{\sum_{i=1}^{n}\left[\left(\sum_{j=1}^{\Upsilon_{\ell_i}} f_{\mathcal{X}_j}(\ell_i)\wp_{III\mathcal{X}_j}(\ell_i)-\overline{f_{\mathcal{X}}}\right)\left(\sum_{j=1}^{\Upsilon_{\ell_i}} f_{\mathcal{Y}_j}(\ell_i)\wp_{III\mathcal{Y}_j}(\ell_i)-\overline{f_{\mathcal{Y}}}\right)\right]}{\sqrt{\sum_{i=1}^{n}\left[\left(\sum_{j=1}^{\Upsilon_{\ell_i}} f_{\mathcal{X}_j}(\ell_i)\wp_{III\mathcal{X}_j}(\ell_i)-\overline{f_{\mathcal{X}}}\right)^2\right]}\sqrt{\sum_{i=1}^{n}\left[\left(\sum_{j=1}^{\Upsilon_{\ell_i}} f_{\mathcal{Y}_j}(\ell_i)\wp_{III\mathcal{Y}_j}(\ell_i)-\overline{f_{\mathcal{Y}}}\right)^2\right]}} = \frac{0.1152}{0.1260} = 0.9146.$$

$$\mathfrak{p}(\mathcal{X},\mathcal{Y}) = \frac{1}{3}\left(\mathfrak{p}_t(\mathcal{X},\mathcal{Y}) + \mathfrak{p}_i(\mathcal{X},\mathcal{Y}) + \mathfrak{p}_f(\mathcal{X},\mathcal{Y})\right),$$

$$\mathfrak{p}(\mathcal{X},\mathcal{Y}) = \frac{1}{3}(0.3901 + 0.2759 + 0.9146),$$

$$\mathfrak{p}(\mathcal{X},\mathcal{Y}) = 0.5269.$$

The CC between the two PSVNHFSs is 0.5269.

Example 3. Let

$$\mathcal{X} = \begin{Bmatrix} \langle\{0.4|0.3,0.6|0.5,0.5|0.2\},\{0.3|0.3,0.5|0.2,0.6|0.4\},\{0.9|0.5,0.7|0.2,0.5|0.3\}\rangle \\ \langle\{0.5|0.4,0.2|0.3,0.1|0.3\},\{0.4|0.3,0.6|0.5,0.3|0.2\},\{0.3|0.4,0.5|0.2,0.4|0.4\}\rangle \\ \langle\{0.5|0.1,0.7|0.4,0.8|0.5\},\{0.4|0.3,0.3|0.4,0.5|0.3\},\{0.8|0.5,0.6|0.2,0.4|0.3\rangle \end{Bmatrix}.$$

And

$$\mathcal{Y} = \begin{Bmatrix} \langle\{0.5|0.1,0.4|0.5,0.2|0.4\},\{0.6|0.3,0.7|0.4,0.4|0.3\},\{0.7|0.4,0.6|0.3,0.4|0.3\}\rangle \\ \langle\{0.6|0.8,0.3|0.2\},\{0.4|0.3,0.2|0.7\},\{0.3|0.7,0.2|0.3\}\rangle \\ \langle\{0.8|0.3,0.7|0.5,0.6|0.2\},\{0.6|0.2,0.3|0.4,0.5|0.4\},\{0.5|0.2,0.7|0.4,0.8|0.4\}\rangle \end{Bmatrix}.$$

Similar to *Example 3*,

$$\overline{t_{\mathcal{X}}} = 0.513, \overline{t_{\mathcal{Y}}} = 0.527, \overline{i_{\mathcal{X}}} = 0.433, \overline{i_{\mathcal{Y}}} = 0.52, \overline{f_{\mathcal{X}}} = 0.637, \overline{f_{\mathcal{Y}}} = 0.56\,.$$

Using *Eqs. (2)-(4)*, we have

$$\mathfrak{p}_t(\mathcal{X},\mathcal{Y}) = \frac{\sum_{i=1}^{n}\left[\left(\sum_{j=1}^{I_{\ell_i}} t_{\mathcal{X}_j}(\ell_i)\wp_{I\mathcal{X}_j}(\ell_i)-\overline{t_{\mathcal{X}}}\right)\left(\sum_{j=1}^{I_{\ell_i}} t_{\mathcal{Y}_j}(\ell_i)\wp_{I\mathcal{Y}_j}(\ell_i)-\overline{t_{\mathcal{Y}}}\right)\right]}{\sqrt{\sum_{i=1}^{n}\left[\left(\sum_{j=1}^{I_{\ell_i}} t_{\mathcal{X}_j}(\ell_i)\wp_{I\mathcal{X}_j}(\ell_i)-\overline{t_{\mathcal{X}}}\right)^2\right]}\sqrt{\sum_{i=1}^{n}\left[\left(\sum_{j=1}^{I_{\ell_i}} t_{\mathcal{Y}_j}(\ell_i)\wp_{I\mathcal{Y}_j}(\ell_i)-\overline{t_{\mathcal{Y}}}\right)^2\right]}} = \frac{0.0354}{0.0838} = 0.4229.$$

$$\mathfrak{p}_i(\mathcal{X},\mathcal{Y}) = \frac{\sum_{i=1}^{n}\left[\left(\sum_{j=1}^{\Lambda_{\ell_i}} i_{\mathcal{X}_j}(\ell_i)\wp_{II\mathcal{X}_j}(\ell_i)-\overline{i_{\mathcal{X}}}\right)\left(\sum_{j=1}^{\Lambda_{\ell_i}} i_{\mathcal{Y}_j}(\ell_i)\wp_{II\mathcal{Y}_j}(\ell_i)-\overline{i_{\mathcal{Y}}}\right)\right]}{\sqrt{\sum_{i=1}^{n}\left[\left(\sum_{j=1}^{\Lambda_{\ell_i}} i_{\mathcal{X}_j}(\ell_i)\wp_{II\mathcal{X}_j}(\ell_i)-\overline{i_{\mathcal{X}}}\right)^2\right]}\sqrt{\sum_{i=1}^{n}\left[\left(\sum_{j=1}^{\Lambda_{\ell_i}} i_{\mathcal{Y}_j}(\ell_i)\wp_{II\mathcal{Y}_j}(\ell_i)-\overline{i_{\mathcal{Y}}}\right)^2\right]}} = \frac{0.0042}{0.0065} = 0.6458.$$

$$\mathfrak{p}_{f}(\mathcal{X},\mathcal{Y})=\frac{\sum_{i=1}^{n}\left[\left(\sum_{j=1}^{\Upsilon_{\ell_i}} f_{\mathcal{X}_j}(\ell_i)\wp_{III\mathcal{X}_j}(\ell_i)-\overline{f_{\mathcal{X}}}\right)\left(\sum_{j=1}^{\Upsilon_{\ell_i}} f_{\mathcal{Y}_j}(\ell_i)\wp_{III\mathcal{Y}_j}(\ell_i)-\overline{f_{\mathcal{Y}}}\right)\right]}{\sqrt{\sum_{i=1}^{n}\left[\left(\sum_{j=1}^{\Upsilon_{\ell_i}} f_{\mathcal{X}_j}(\ell_i)\wp_{III\mathcal{X}_j}(\ell_i)-\overline{f_{\mathcal{X}}}\right)^2\right]}\sqrt{\sum_{i=1}^{n}\left[\left(\sum_{j=1}^{\Upsilon_{\ell_i}} f_{\mathcal{Y}_j}(\ell_i)\wp_{III\mathcal{Y}_j}(\ell_i)-\overline{f_{\mathcal{Y}}}\right)^2\right]}}=\frac{0.1129}{0.1281}=$$

$0.8812.$

$$\mathfrak{p}(\mathcal{X},\mathcal{Y})=\frac{1}{3}\left(\mathfrak{p}_{t}(\mathcal{X},\mathcal{Y})+\mathfrak{p}_{i}(\mathcal{X},\mathcal{Y})+\mathfrak{p}_{f}(\mathcal{X},\mathcal{Y})\right),$$

$$\mathfrak{p}(\mathcal{X},\mathcal{Y})=\frac{1}{3}\left(\mathfrak{p}_{t}(\mathcal{X},\mathcal{Y})+\mathfrak{p}_{i}(\mathcal{X},\mathcal{Y})+\mathfrak{p}_{f}(\mathcal{X},\mathcal{Y})\right),$$

$$\mathfrak{p}(\mathcal{X},\mathcal{Y})=0.65.$$

The CC between the two PSVNHFSs is 0.65.

3.2 | Proposed Weighted CC between Two PSVNHFSs

The TMHD, IMHD and FMHD are assumed to be equally significant and given equal status in the calculation above. But in reality, different elements frequently have differing scales of significance, so they ought to be given differing weights correspondingly. The context and particular goals of the decision-making process determine how these degrees are weighted:

I. The degree of truth needs to be given more weight if the goal is to maximize the certainty or honesty of a choice. This is typical when an option's authenticity or dependability is crucial.

II. The degree of indeterminacy should be prioritized if reducing uncertainty or indeterminacy is the goal. This is significant in situations where clarity and less ambiguity are essential.

III. The degree of falsity should be given more weight if the goal is to reduce the falsehood or incorrectness. This is essential in situations where avoiding mistakes or incorrect choices is crucial.

IV. The choice criteria may determine the degree that should be given priority. For example, the degree of indeterminacy may be critical in risk assessment, whereas the degree of truth may be more significant in quality control.

To deal with such situations, we assign the weights $\omega_i > 0$ to each element of X such that $\sum_{i=1}^{n}\omega_i = 1$ and define the weighted CC between two PSVNHFSs $\mathcal{X}$ and $\mathcal{Y}$ as follows:

$$\mathfrak{p}_{\omega}(\mathcal{X},\mathcal{Y})=\left[\frac{1}{\zeta+2}*\mathfrak{p}_{\omega t}(\mathcal{X},\mathcal{Y})+\frac{\zeta}{\zeta+2}*\mathfrak{p}_{\omega i}(\mathcal{X},\mathcal{Y})+\frac{1}{\zeta+2}*\mathfrak{p}_{\omega f}(\mathcal{X},\mathcal{Y})\right], \tag{14}$$

where ζ is the risk preference coefficient. This coefficient can be adjusted as follows:

I. ζ =1: neutral towards risk, i.e., equal weight is given to all three CCs.

II. ζ >1: risk-averse, i.e., more weight is given to reducing indeterminacy and falsity, reflecting a preference for certainty and reliability.

III. ζ <1: risk-seeking, i.e., more weight is given to increasing truth, reflecting a willingness to accept higher indeterminacy and falsity for potential gains in truth.

IV. The risk preference coefficient directly affects the CC for TMHD and FMHD. A higher ζ reduces the weight of truth, reflecting risk aversion, while IMHD is Proportional to ζ, i.e., a higher ζ Increases the weight of indeterminacy, emphasizing the need to minimize uncertainty.

Where

$$\mathfrak{p}_{\omega t}(\mathcal{X},\mathcal{Y})=\frac{\sum_{i=1}^{n}\left[\left(\omega_i\sum_{j=1}^{I_{\ell_i}} t_{\mathcal{X}_j}(\ell_i)\wp_{I\mathcal{X}_j}(\ell_i)-\overline{t_{\omega\mathcal{X}}}\right)\left(\omega_i\sum_{j=1}^{I_{\ell_i}} t_{\mathcal{Y}_j}(\ell_i)\wp_{I\mathcal{Y}_j}(\ell_i)-\overline{t_{\omega\mathcal{Y}}}\right)\right]}{\sqrt{\sum_{i=1}^{n}\left[\left(\omega_i\sum_{j=1}^{I_{\ell_i}} t_{\mathcal{X}_j}(\ell_i)\wp_{I\mathcal{X}_j}(\ell_i)-\overline{t_{\omega\mathcal{X}}}\right)^2\right]}\sqrt{\sum_{i=1}^{n}\left[\left(\omega_i\sum_{j=1}^{I_{\ell_i}} t_{\mathcal{Y}_j}(\ell_i)\wp_{I\mathcal{Y}_j}(\ell_i)-\overline{t_{\omega\mathcal{Y}}}\right)^2\right]}}, \tag{15}$$

$$\mathfrak{p}_{\omega i}(\mathcal{X},\mathcal{Y}) = \frac{\sum_{i=1}^{n}\left[\left(\omega_i \sum_{j=1}^{\Lambda_{\ell_i}} i_{\mathcal{X}_j}(\ell_i)\wp_{II\mathcal{X}_j}(\ell_i) - \overline{\imath_{\mathcal{X}}}\right)\left(\omega_i \sum_{j=1}^{\Lambda_{\ell_i}} i_{\mathcal{Y}_j}(\ell_i)\wp_{II\mathcal{Y}_j}(\ell_i) - \overline{\imath_{\mathcal{Y}}}\right)\right]}{\sqrt{\sum_{i=1}^{n}\left[\left(\omega_i \sum_{j=1}^{\Lambda_{\ell_i}} i_{\mathcal{X}_j}(\ell_i)\wp_{II\mathcal{X}_j}(\ell_i) - \overline{\imath_{\mathcal{X}}}\right)^2\right]}\sqrt{\sum_{i=1}^{n}\left[\left(\omega_i \sum_{j=1}^{\Lambda_{\ell_i}} i_{\mathcal{Y}_j}(\ell_i)\wp_{II\mathcal{Y}_j}(\ell_i) - \overline{\imath_{\mathcal{Y}}}\right)^2\right]}}, \quad (16)$$

$$\mathfrak{p}_{\omega f}(\mathcal{X},\mathcal{Y}) = \frac{\sum_{i=1}^{n}\left[\left(\omega_i \sum_{j=1}^{\Upsilon_{\ell_i}} f_{\mathcal{X}_j}(\ell_i)\wp_{III\mathcal{X}_j}(\ell_i) - \overline{f_{\mathcal{X}}}\right)\left(\omega_i \sum_{j=1}^{\Upsilon_{\ell_i}} f_{\mathcal{Y}_j}(\ell_i)\wp_{III\mathcal{Y}_j}(\ell_i) - \overline{f_{\mathcal{Y}}}\right)\right]}{\sqrt{\sum_{i=1}^{n}\left[\left(\omega_i \sum_{j=1}^{\Upsilon_{\ell_i}} f_{\mathcal{X}_j}(\ell_i)\wp_{III\mathcal{X}_j}(\ell_i) - \overline{f_{\mathcal{X}}}\right)^2\right]}\sqrt{\sum_{i=1}^{n}\left[\left(\omega_i \sum_{j=1}^{\Upsilon_{\ell_i}} f_{\mathcal{Y}_j}(\ell_i)\wp_{III\mathcal{Y}_j}(\ell_i) - \overline{f_{\mathcal{Y}}}\right)^2\right]}}. \quad (17)$$

The weighted mean of the TMHD of PSVNHFS $\mathcal{X}$ and $\mathcal{Y}$ are

$$\overline{t_{\omega\mathcal{X}}} = \frac{1}{n}\sum_{i=1}^{n}\omega_i\left(\sum_{j=1}^{I_{\ell_i}} t_{\mathcal{X}_j}(\ell_i)\,\wp_{I\mathcal{X}_j}(\ell_i)\right). \quad (18)$$

$$\overline{t_{\omega\mathcal{Y}}} = \frac{1}{n}\sum_{i=1}^{n}\omega_i\left(\sum_{j=1}^{I_{\ell_i}} t_{\mathcal{Y}_j}(\ell_i)\,\wp_{I\mathcal{Y}_j}(\ell_i)\right). \quad (19)$$

The weighted mean of the IMHD of PSVNHFS $\mathcal{X}$ and $\mathcal{Y}$ are

$$\overline{\imath_{\omega\mathcal{X}}} = \frac{1}{n}\sum_{i=1}^{n}\omega_i\left(\sum_{j=1}^{\Lambda_{\ell_i}} i_{\mathcal{X}_j}(\ell_i)\,\wp_{II\mathcal{X}_j}(\ell_i)\right). \quad (20)$$

$$\overline{\imath_{\omega\mathcal{Y}}} = \frac{1}{n}\sum_{i=1}^{n}\omega_i\left(\sum_{j=1}^{\Lambda_{\ell_i}} i_{\mathcal{Y}_j}(\ell_i)\,\wp_{II\mathcal{Y}_j}(\ell_i)\right). \quad (21)$$

The weighted mean of the FMHD of PSVNHFS $\mathcal{X}$ and $\mathcal{Y}$ are

$$\overline{f_{\omega\mathcal{X}}} = \frac{1}{n}\sum_{i=1}^{n}\omega_i\left(\sum_{j=1}^{\Upsilon_{\ell_i}} f_{\mathcal{X}_j}(\ell_i)\,\wp_{III\mathcal{X}_j}(\ell_i)\right). \quad (22)$$

$$\overline{f_{\omega\mathcal{Y}}} = \frac{1}{n}\sum_{i=1}^{n}\omega_i\left(\sum_{j=1}^{\Upsilon_{\ell_i}} f_{\mathcal{Y}_j}(\ell_i)\,\wp_{III\mathcal{Y}_j}(\ell_i)\right). \quad (23)$$

Theorem 2. Let $\mathcal{X} = \langle t_{\mathcal{X}}|\wp_{I\mathcal{X}}, i_{\mathcal{X}}|\wp_{II\mathcal{X}}, f_{\mathcal{X}}|\wp_{III\mathcal{X}}\rangle$ and $\mathcal{Y} = \langle t_{\mathcal{Y}}|\wp_{I\mathcal{Y}}, i_{\mathcal{Y}}|\wp_{II\mathcal{Y}}, f_{\mathcal{Y}}|\wp_{III\mathcal{Y}}\rangle$ be two PSVNHFS defined on $X = (\ell_1, \ell_2, \dots, \ell_n)$, then the WCC between $\mathcal{X}$ and $\mathcal{Y}$ will satisfy the following conditions:

I. $\mathfrak{p}_{\omega}(\mathcal{X},\mathcal{Y}) = \mathfrak{p}_{\omega}(\mathcal{Y},\mathcal{X})$.

II. $\mathfrak{p}_{\omega}(\mathcal{X},\mathcal{Y}) = 1$ if $\mathcal{X} = \mathcal{Y}$.

III. $-1 \le \mathfrak{p}_{\omega}(\mathcal{X},\mathcal{Y}) \le 1$.

Proof: it can easily be proved by interchanging $\mathcal{X}$ to $\mathcal{Y}$ in $\mathfrak{p}_{\omega t}(\mathcal{X},\mathcal{Y}), \mathfrak{p}_{\omega i}(\mathcal{X},\mathcal{Y})$ and $\mathfrak{p}_{\omega f}(\mathcal{X},\mathcal{Y})$ and substituting the obtained values in values in $\mathfrak{p}_{\omega}(\mathcal{X},\mathcal{Y})$. We will get

$$\mathfrak{p}_{\omega}(\mathcal{Y},\mathcal{X}) = \left[\frac{1}{\zeta+2} * \mathfrak{p}_{\omega t}(\mathcal{Y},\mathcal{X}) + \frac{\zeta}{\zeta+2} * \mathfrak{p}_{\omega i}(\mathcal{Y},\mathcal{X}) + \frac{1}{\zeta+2} * \mathfrak{p}_{\omega f}(\mathcal{Y},\mathcal{X})\right] = \mathfrak{p}_{\omega}(\mathcal{X},\mathcal{Y}).$$

Proof: if $\mathcal{X} = \mathcal{Y}$, then from *Eq. (15)*

$$\mathfrak{p}_{\omega t}(\mathcal{X},\mathcal{X}) = \frac{\sum_{i=1}^{n}\left[\left(\omega_i \sum_{j=1}^{I_{\ell_i}} t_{\mathcal{X}_j}(\ell_i)\wp_{I\mathcal{X}_j}(\ell_i) - \overline{t_{\mathcal{X}}}\right)\left(\omega_i \sum_{j=1}^{I_{\ell_i}} t_{\mathcal{X}_j}(\ell_i)\wp_{I\mathcal{X}_j}(\ell_i) - \overline{t_{\mathcal{X}}}\right)\right]}{\sqrt{\sum_{i=1}^{n}\left[\left(\omega_i \sum_{j=1}^{I_{\ell_i}} t_{\mathcal{X}_j}(\ell_i)\wp_{I\mathcal{X}_j}(\ell_i) - \overline{t_{\mathcal{X}}}\right)^2\right]}\sqrt{\sum_{i=1}^{n}\left[\left(\omega_i \sum_{j=1}^{I_{\ell_i}} t_{\mathcal{X}_j}(\ell_i)\wp_{I\mathcal{X}_j}(\ell_i) - \overline{t_{\mathcal{X}}}\right)^2\right]}}.$$

$$\mathfrak{p}_{\omega t}(\mathcal{X},\mathcal{X}) = \frac{\sum_{i=1}^{n}\left[\left(\omega_i \sum_{j=1}^{I_{\ell_i}} t_{\mathcal{X}_j}(\ell_i)\wp_{I\mathcal{X}_j}(\ell_i) - \overline{t_{\mathcal{X}}}\right)^2\right]}{\sum_{i=1}^{n}\left[\left(\omega_i \sum_{j=1}^{I_{\ell_i}} t_{\mathcal{X}_j}(\ell_i)\wp_{I\mathcal{X}_j}(\ell_i) - \overline{t_{\mathcal{X}}}\right)^2\right]} = 1.$$

Similarly, $\mathfrak{p}_{\omega i}(\mathcal{X},\mathcal{X}) = \mathfrak{p}_{\omega f}(\mathcal{X},\mathcal{X}) = 1$.

On putting these values in *Eq. (14)*, we have

$$\mathfrak{p}_{\omega}(\mathcal{X},\mathcal{X})=\frac{1}{3}\left(\frac{1}{\zeta+2}*\mathfrak{p}_{\omega t}(\mathcal{X},\mathcal{X})+\frac{\zeta}{\zeta+2}*\mathfrak{p}_{\omega i}(\mathcal{X},\mathcal{X})+\frac{1}{\zeta+2}*\mathfrak{p}_{\omega f}(\mathcal{X},\mathcal{X})\right)=1.$$

Similarly, we can prove $\mathfrak{p}(\mathcal{Y},\mathcal{Y})=1$.

Proof: from Cauchy Schwarz inequality, we know that

$$(u_1v_1+u_2v_2+\cdots+u_nv_n)^2\le(u_1^2+u_2^2+\cdots+u_n^2)(v_1^2+v_2^2+\cdots+v_n^2)\text{ for all } u_i,v_i\in\Re.$$

Using the inequality in *Eq. (15)*, we have

$$\left\{\sum_{i=1}^{n}\left[\left(\omega_i\sum_{j=1}^{I_{\ell_i}}t_{\mathcal{X}_j}(\ell_i)\,p_{I\mathcal{X}_j}(\ell_i)-\overline{t_{\mathcal{X}}}\right)\left(\omega_i\sum_{j=1}^{I_{\ell_i}}t_{\mathcal{Y}_j}(\ell_i)\,p_{I\mathcal{Y}_j}(\ell_i)-\overline{t_{\mathcal{Y}}}\right)\right]\right\}^2\le$$
$$\sum_{i=1}^{n}\left[\left(\omega_i\sum_{j=1}^{I_{\ell_i}}t_{\mathcal{X}_j}(\ell_i)\,p_{I\mathcal{X}_j}(\ell_i)-\overline{t_{\mathcal{X}}}\right)^2\right]\sum_{i=1}^{n}\left[\left(\omega_i\sum_{j=1}^{I_{\ell_i}}t_{\mathcal{Y}_j}(\ell_i)\,p_{I\mathcal{Y}_j}(\ell_i)-\overline{t_{\mathcal{Y}}}\right)^2\right].$$

On taking square root on both sides, we have

$$\left|\sum_{i=1}^{n}\left[\left(\omega_i\sum_{j=1}^{I_{\ell_i}}t_{\mathcal{X}_j}(\ell_i)\,p_{I\mathcal{X}_j}(\ell_i)-\overline{t_{\mathcal{X}}}\right)\left(\omega_i\sum_{j=1}^{I_{\ell_i}}t_{\mathcal{Y}_j}(\ell_i)\,p_{I\mathcal{Y}_j}(\ell_i)-\overline{t_{\mathcal{Y}}}\right)\right]\right|$$
$$\le\sqrt{\sum_{i=1}^{n}\left[\left(\omega_i\sum_{j=1}^{I_{\ell_i}}t_{\mathcal{X}_j}(\ell_i)\,p_{I\mathcal{X}_j}(\ell_i)-\overline{t_{\mathcal{X}}}\right)^2\right]}\sqrt{\sum_{i=1}^{n}\left[\left(\omega_i\sum_{j=1}^{I_{\ell_i}}t_{\mathcal{X}_j}(\ell_i)\,p_{I\mathcal{X}_j}(\ell_i)-\overline{t_{\mathcal{X}}}\right)^2\right]}\;. \tag{24}$$

Which implies

$$\frac{\left|\sum_{i=1}^{n}\left[\left(\omega_i\sum_{j=1}^{I_{\ell_i}}t_{\mathcal{X}j}(\ell_i)p_{I\mathcal{X}j}(\ell_i)-\overline{t_{\mathcal{X}}}\right)\left(\omega_i\sum_{j=1}^{I_{\ell_i}}t_{\mathcal{Y}j}(\ell_i)p_{I\mathcal{Y}j}(\ell_i)-\overline{t_{\mathcal{Y}}}\right)\right]\right|}{\sqrt{\sum_{i=1}^{n}\left[\left(\omega_i\sum_{j=1}^{I_{\ell_i}}t_{\mathcal{X}j}(\ell_i)p_{I\mathcal{X}j}(\ell_i)-\overline{t_{\mathcal{X}}}\right)^2\right]}\sqrt{\sum_{i=1}^{n}\left[\left(\omega_i\sum_{j=1}^{I_{\ell_i}}t_{\mathcal{Y}j}(\ell_i)p_{I\mathcal{Y}j}(\ell_i)-\overline{t_{\mathcal{Y}}}\right)^2\right]}}\le 1.$$

Hence, $|\mathfrak{p}_{\omega t}(\mathcal{X},\mathcal{Y})|\le 1$. Similarly, we can show $|\mathfrak{p}_{\omega i}(\mathcal{X},\mathcal{Y})\le 1|$ and $|\mathfrak{p}_{\omega f}(\mathcal{X},\mathcal{Y})\le 1|$.

Further,

$$|\mathfrak{p}_{\omega}(\mathcal{X},\mathcal{Y})|=\frac{1}{3}\left(\left|\frac{1}{\zeta+2}*\mathfrak{p}_{\omega t}(\mathcal{X},\mathcal{Y})\right|+\left|\frac{\zeta}{\zeta+2}*\mathfrak{p}_{\omega i}(\mathcal{X},\mathcal{Y})\right|+\left|\frac{1}{\zeta+2}*\mathfrak{p}_{\omega f}(\mathcal{X},\mathcal{Y})\right|\right).$$

Hence prove.

$$|\mathfrak{p}_{\omega}(\mathcal{X},\mathcal{Y})|\le\frac{1}{3}*\frac{1}{\zeta+2}+\frac{1}{3}*\frac{\zeta}{\zeta+2}+\frac{1}{3}*\frac{1}{\zeta+2},$$
$$|\mathfrak{p}_{\omega}(\mathcal{X},\mathcal{Y})|\le 1\;\rightarrow\;-1\le\mathfrak{p}_{\omega}(\mathcal{X},\mathcal{Y})\le 1. \tag{25}$$

4 | Decision-Making Algorithm based on the WCC for PSVNHFSs

We use the developed CC and WCC measures to find the best alternatives in MADM based on PSVNHFS. For a MADM problem, let $\mathcal{F}=\{\mathcal{F}_1,\mathcal{F}_2,\dots,\mathcal{F}_g)$ are a discrete set of alternatives, and $\mathcal{J}=\{\mathcal{J}_1,\mathcal{J}_2,\dots\mathcal{J}_v\}$ are a set of attributes. The $\omega=\{\omega_1,\omega_2\dots,\omega_r)'$ are the weight vector of attributes such that $0\le\omega_j\le 1$ and $\sum_{j=1}^{v}\omega_j=1$. If the DMs anonymously provide several values for the alternative $\mathcal{F}$ under the attribute $\wp$, and each value has precise probabilistic information based on specific rules, then these values can be considered PSVNHFS $\mathcal{L}_{ij}=\langle t_{ij}|p_{Iij},i_{ij}|p_{IIij},f_{ij}|p_{IIIij}\rangle$. Suppose the decision matrix $\mathcal{L}=[\mathcal{L}_{ij}]_{g\times v}$ is the PSVNHFS decision matrix where $i=1,2,\dots g$ and $j=1,2,\dots,v$ based on the response of decision makers. The following algorithm is used in decision-making based on the decision matrix:

Step 1. Apply the following equation to normalize the decision matrices.

$$\mathcal{L}_{ij} = \begin{cases} \langle t_{ij}|p_{Iij}, i_{ij}|p_{IIij}, f_{ij}|p_{IIIij}\rangle \text{ if } \wp_j \in I_1 \\ \langle f_{ij}|p_{IIIij}, i_{ij}|p_{IIij}, t_{ij}|p_{Iij}\rangle \text{ if } \wp_j \in I_2 \end{cases}, \tag{26}$$

where I_1 represents the positive type and I_2 represents the negative type.

Step 2. Construct a new matrix $\mathcal{R} = [r_{ij}]_{g\times v}$ based on the equation given in *Step 1*

$$\mathcal{R} = [r_{ij}]_{g\times v} = \begin{bmatrix} r_{11} & r_{12} & \dots & r_{1v} \\ r_{21} & r_{22} & \dots & r_{2v} \\ \vdots & \vdots & \ddots & \vdots \\ r_{g1} & r_{g2} & \dots & r_{gv} \end{bmatrix}. \tag{27}$$

Step 3. Obtain the value of the weights ω_j such that $0 \le \omega_j \le 1$ and $\sum_{j=1}^{v} \omega_j = 1$.

Step 4. Find the positive ideal solution based on the PSVNHFS using the below equations:

$$\mathcal{F}^+ = [\mathcal{J}_j^+]_{1\times v} = \left[\underbrace{\max}_{i} a_{ij}\right]_{1\times v}.$$

Step 5. Calculate the WCC between $\mathcal{F}_i$ and $\mathcal{F}^+$ for all $i = 1,2,\dots g$ using *Eq. (14)*.

Step 6. Determine the final ranking of all alternatives $\mathfrak{p}_\omega(\mathcal{F}_i, \mathcal{F}^+)$ for all $i = 1,2,\dots g$ from highest to lowest. The alternative with the maximum CC is the optimal choice.

5 | MADM Method Utilizing the WCC in SCM

In a manufacturing company, the choice of a supplier can significantly impact production efficiency, cost, product quality and overall operational success. Choosing the right supplier involves evaluating multiple factors affecting the company's short-term and long-term performance. The goal is to choose the best supplier from a list of potential candidates. The selection criteria include several key attributes that affect the company's decision. Each attribute has inherent uncertainty, represented using probabilistic neutrosophic hesitant sets. The company wants to find the Supplier that best meets its needs while considering these uncertainties. Let us consider four potential suppliers $(\mathcal{F}_1, \mathcal{F}_2, \mathcal{F}_3, \mathcal{F}_4)$ and five attributes $(\sigma_1, \sigma_2, \sigma_3, \sigma_4, \sigma_5)$, which are quality, cost, delivery time, reliability and environmental impact, as given in *Fig. 1*. The decision matrix based on suppliers and attributes is given in *Table 1*.

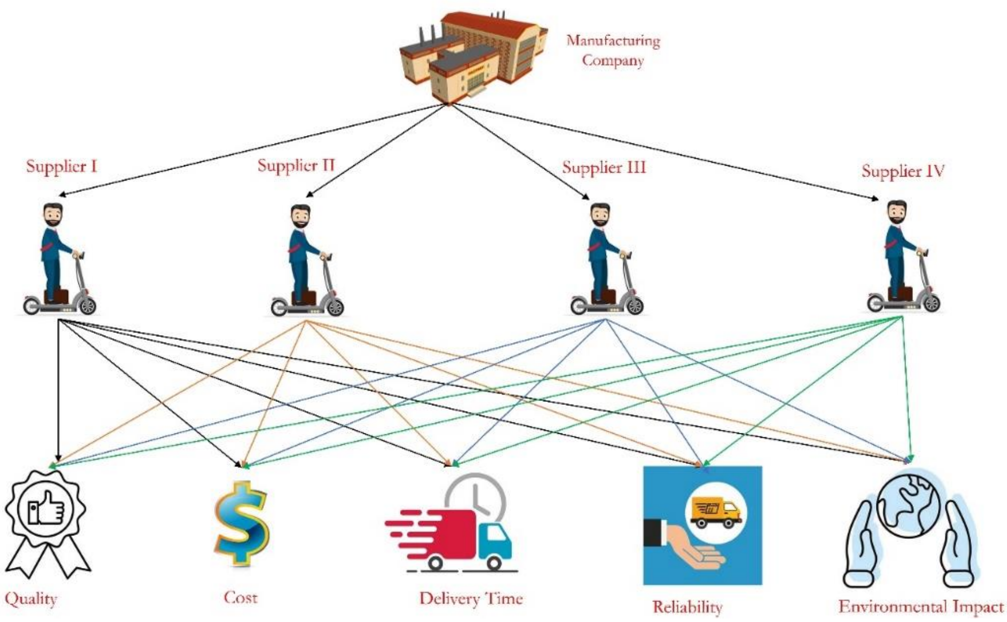

Fig. 1. Tree diagram of the suppliers and attributes.

Table 1. Decision matrix in the form of PSVNHFS.

<table>
<tr><th>Attributes</th><th>Supplier I ($\mathcal{F}_1$)</th><th>Supplier II ($\mathcal{F}_2$)</th></tr>
<tr><td>Quality (σ_1)</td><td>⟨{0.49|0.42,0.45|0.42,0.42|0.16}
{0.83|0.41,0.81|0.48,0.7|0.11}
{0.25|0.46,0.21|0.22,0.18|0.32}⟩</td><td>⟨{0.35|0.54,0.28|0.15,0.25|0.31}
{0.92|0.47,0.91|0.43,0.87|0.1}
{0.61|0.3,0.59|0.14,0.5|0.56}⟩</td></tr>
<tr><td>Cost (σ_2)</td><td>⟨{0.20|0.34,0.18|0.13,0.17|0.53}
{0.60|0.48,0.51|0.49,0.48|0.03}
{0.78|0.46,0.71|0.42,0.67|0.12}⟩</td><td>⟨{0.83|0.55,0.76|0.16,0.66|0.29}
{0.46|0.34,0.37|0.48,0.33|0.18}
{0.87|0.04,0.83|0.65,0.82|0.31}⟩</td></tr>
<tr><td>Delivery Time (σ_3)</td><td>⟨{0.21|0.23,0.16|0.40,0.08|0.37}
{0.34|0.75,0.30|0.25}
{0.14|0.56,0.11|0.26,0.08|0.18}⟩</td><td>⟨{0.58|0.56,0.46|0.17,0.41|0.27}
{0.94|0.28,0.92|0.54,0.84|0.18}
{0.63|0.62,0.61|0.24,0.52|0.14}⟩</td></tr>
<tr><td>Reliability (σ_4)</td><td>⟨{0.39|0.04,0.34|0.43,0.31|0.53}
{0.78|0.35,0.66|0.32,0.62|0.33}
{0.23|0.06,0.13|0.61,0.08|0.33}⟩</td><td>⟨{0.17|0.16,0.15|0.37,0.04|0.47}
{0.33|0.19,0.28|0.23,0.22|0.58}
{0.92|0.43,0.87|0.28,0.79|0.29, }⟩</td></tr>
<tr><td>Environmental Impact (σ_5)</td><td>⟨{0.39|0.32,0.38|0.08,0.31|0.60}
{0.72|0.64,0.68|0.36}
{0.15|0.53,0.04|0.18,0.03|0.29}⟩</td><td>⟨{0.37|0.52,0.21|0.48}
{0.76|0.82,0.74|0.18}
{0.84|0.8,0.74|0.19,0.72|0.01}⟩</td></tr>
<tr><th>Attributes</th><th>Supplier III ($\mathcal{F}_3$)</th><th>Supplier IV ($\mathcal{F}_4$)</th></tr>
<tr><td>Quality (σ_1)</td><td>⟨{0.58|0.55,0.57|0.19,0.48|0.26}
{0.77|0.09,0.73|0.91}
{0.41|0.27,0.29|0.34,0.26|0.38}⟩</td><td>⟨{0.64|0.17,0.59|0.87}
{0.72|0.39,0.68|0.26,0.58|0.35}
{0.35|0.23,0.27|0.57,0.2|0.2}⟩</td></tr>
<tr><td>Cost (σ_2)</td><td>⟨{0.52|0.71,0.5|0.27,0.44|0.02}
{0.71|0.01,0.7|. 28,0.55|0.71}
{0.94|0.44,0.83|0.56}⟩</td><td>⟨{0.83|0.43,0.78|0.42,0.7|0.15}
{0.47|0.96,0.43|0.04}
{0.57|0.1,0.53|0.86,0.44|0.4}⟩</td></tr>
<tr><td>Delivery Time (σ_3)</td><td>⟨{0.79|0.26,0.74|0.43,0.71|0.32}
{0.28|0.41,0.2|0.24,0.11|0.35}
{0.7|0.33,0.63|0.33,0.62|0.35}⟩</td><td>⟨{0.66|0.39,0.58|0.28,0.55|0.33}
{0.94|0.19,0.93|0.28,0.83|0.53}
{0.15|0.71,0.03|0.29}⟩</td></tr>
<tr><td>Reliability (σ_4)</td><td>⟨{0.89|0.37,0.83|0.63}
{0.79|0.03,0.78|0.54,0.73|0.43}
{0.55|0.67,0.49|0.03,0.44|0.3}⟩</td><td>⟨{0.79|0.33,0.71|0.28,0.68|0.39}
{0.79|0.19,0.7|0.37,0.68|0.44}
{0.48|0.37,0.45|0.42,0.37|0.21}⟩</td></tr>
<tr><td>Environmental Impact (σ_5)</td><td>⟨{0.26|0.11,0.21|0.82,0.14|0.07}
{0.57|0.18,0.55|0.28,0.53|0.54}
{0.72|0.53,0.69|0.01,0.65|0.46}⟩</td><td>⟨{0.78|0.16,0.7|0.44,0.62|0.40}
{0.49|0.43,0.33|0.43,0.31|0.14}
{0.59|0.19,0.55|0.38,0.52|0.43}⟩</td></tr>
</table>

Now, we will utilize the proposed WCC measure to select the best Supplier considering the given attributes.

Step 1. Since all the attributes are derived from I_1, normalization is unnecessary.

Step 2. The attributes are already on a consistent scale, making additional normalization redundant. Therefore, we can proceed with the analysis without performing any normalization.

Step 3. The positive ideal Solution based on the *Table 1* is

$$\mathcal{J}^+ = (\sigma_1{}^+, \sigma_2{}^+, \sigma_3{}^+, \sigma_4{}^+, \sigma_5{}^+) = \begin{pmatrix} \left\langle \begin{matrix} 0.64|0.17,0.59|0.87 \\ 0.72|0.39,0.68|0.26,0.58|0.35 \\ \{0.35|0.23,0.27|0.57,0.2|0.2\} \end{matrix} \right\rangle, & \left\langle \begin{matrix} \{0.83|0.55,0.76|0.16,0.66|0.29\} \\ 0.46|0.34,0.37|0.48,0.33|0.18 \\ \{0.87|0.04,0.83|0.65,0.82|0.31\} \end{matrix} \right\rangle, \\ \left\langle \begin{matrix} \{0.79|0.26,0.74|0.43,0.71|0.32\} \\ \{0.28|0.41,0.2|0.24,0.11|0.35\} \\ \{0.7|0.33,0.63|0.33,0.62|0.35\} \end{matrix} \right\rangle, & \left\langle \begin{matrix} 0.89|0.37,0.83|0.63 \\ \{0.79|0.03,0.78|0.54,0.73|0.43\} \\ \{0.55|0.67,0.49|0.03,0.44|0.3\} \end{matrix} \right\rangle, \\ \multicolumn{2}{c}{\left\langle \begin{matrix} \{0.78|0.16,0.7|0.44,0.62|0.40\} \\ \{0.49|0.43,0.33|0.43,0.31|0.14\} \\ \{0.59|0.19,0.55|0.38,0.52|0.43\} \end{matrix} \right\rangle} \end{pmatrix}.$$

Step 4. Let the weighted vector of attributes be $\omega = (0.35,0.25,0.20,0.15,0.05)'$.

Step 5. Calculate the WCCs $\mathfrak{p}_{\omega t}(\mathcal{F}_i, \mathcal{F}^+)$, $\mathfrak{p}_{\omega i}(\mathcal{F}_i, \mathcal{F}^+)$, $\mathfrak{p}_{\omega \mathfrak{f}}(\mathcal{F}_i, \mathcal{F}^+)$ and $\mathfrak{p}_{\omega}(\mathcal{F}_i, \mathcal{F}^+)$ between $\mathcal{F}_i$ and $\mathcal{F}^+$ for all $i = 1,2,3,4$ using *Eq. (14)* where ζ =1. The WCCs given in *Table 2*.

Table 2. The WCCs for $\mathfrak{p}_{\omega t}(\mathcal{F}_i, \mathcal{F}^+)$, $\mathfrak{p}_{\omega i}(\mathcal{F}_i, \mathcal{F}^+)$, $\mathfrak{p}_{\omega \mathfrak{f}}(\mathcal{F}_i, \mathcal{F}^+)$ and $\mathfrak{p}_{\omega}(\mathcal{F}_i, \mathcal{F}^+)$.

WCCs	$\mathcal{F}_1$	$\mathcal{F}_2$	$\mathcal{F}_3$	$\mathcal{F}_4$
$\mathfrak{p}_{\omega t}(\mathcal{F}_i, \mathcal{F}^+)$	0.6962	0.7498	0.9296	0.9794
$\mathfrak{p}_{\omega i}(\mathcal{F}_i, \mathcal{F}^+)$	0.8095	0.7226	0.807	0.8702
$\mathfrak{p}_{\omega \mathfrak{f}}(\mathcal{F}_i, \mathcal{F}^+)$	0.6511	0.9603	0.7175	0.6524
$\mathfrak{p}_{\omega}(\mathcal{F}_i, \mathcal{F}^+)$	0.7189	0.8109	0.8180	0.8340

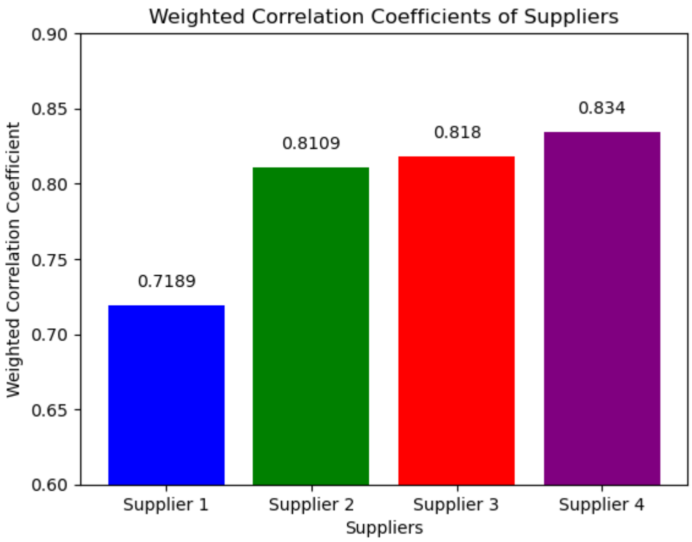

Fig. 2. Bar chart of the WCC corresponding to suppliers.

Step 6. From the obtained WCCs corresponding to each Supplier in *Table 2* and *Fig. 2*, The ranking of the four suppliers is $\mathcal{F}_4 \succ \mathcal{F}_3 \succ \mathcal{F}_2 \succ \mathcal{F}_1$. So, when the weight of the quality is 35%, the cost is 25%, the delivery Time is 20%, the reliability is 15%, the environmental impact is 5%, and the decision maker's attitude towards risk in the supplier selection is neutral then the supplier 4 ($\mathcal{F}_4$) is the best choice, balancing high quality with acceptable costs and good delivery reliability.

5.1 | Assessment of Parameters Effect on Ranking

5.1.1 | Assessment of weights effect on ranking with constant risk preferenceto

Assess the effect of weights on the supplier ranking, we can analyze how changes in the attribute weights influence the WCCs and rankings of the suppliers. We'll consider different weight scenarios at ζ =1. The weights are generated using Python and given below:

Weight Set 1: [0.133,0.338,0.261,0.213,0.055].

Weight Set 2: [0.065,0.024,0.363,0.252,0.296].

Weight Set 3: [0.009,0.437,0.375,0.096,0.083].

Weight Set 4: [0.106,0.175,0.302,0.249,0.168].

Weight Set 5: [0.328,0.075,0.157,0.196,0.244].

Weight Set 6: [0.367,0.093,0.241,0.277,0.022].

Weight Set 7: [0.22,0.062,0.024,0.344,0.35].

Weight Set 8: [0.346,0.13,0.042,0.293,0.188].

Table 3. Ranking of suppliers with different weights of attributes and neutral risk.

Weights ($\boldsymbol{\omega}$)	$\mathfrak{p}_\omega(\mathcal{F}_1,\mathcal{F}^+)$	$\mathfrak{p}_\omega(\mathcal{F}_2,\mathcal{F}^+)$	$\mathfrak{p}_\omega(\mathcal{F}_3,\mathcal{F}^+)$	$\mathfrak{p}_\omega(\mathcal{F}_4,\mathcal{F}^+)$	Ranking
$\omega_1 = [0.133,0.338,0.261,0.213,0.055]'$	0.690	0.753	0.856	0.851	$\mathcal{F}_3 \succ \mathcal{F}_4 \succ \mathcal{F}_2 \succ \mathcal{F}_1$
$\omega_2 = [0.065,0.024,0.363,0.252,0.296]'$	0.832	0.835	0.825	0.827	$\mathcal{F}_2 \succ \mathcal{F}_1 \succ \mathcal{F}_4 \succ \mathcal{F}_3$
$\omega_3 = [0.009,0.437,0.375,0.096,0.083]'$	0.891	0.921	0.917	0.876	$\mathcal{F}_2 \succ \mathcal{F}_3 \succ \mathcal{F}_1 \succ \mathcal{F}_4$
$\omega_4 = [0.106,0.175,0.302,0.249,0.168]'$	0.273	0.597	0.699	0.676	$\mathcal{F}_3 \succ \mathcal{F}_4 \succ \mathcal{F}_2 \succ \mathcal{F}_1$
$\omega_5 = [0.328,0.075,0.157,0.196,0.244]'$	0.635	0.621	0.708	0.808	$\mathcal{F}_4 \succ \mathcal{F}_3 \succ \mathcal{F}_1 \succ \mathcal{F}_2$
$\omega_6 = [0.367,0.093,0.241,0.277,0.022]'$	0.680	0.715	0.896	0.898	$\mathcal{F}_4 \succ \mathcal{F}_3 \succ \mathcal{F}_2 \succ \mathcal{F}_1$
$\omega_7 = [0.22,0.062,0.024,0.344,0.35]'$	0.794	0.676	0.894	0.953	$\mathcal{F}_4 \succ \mathcal{F}_3 \succ \mathcal{F}_1 \succ \mathcal{F}_2$
$\omega_8 = [0.346,0.13,0.042,0.293,0.188]'$	0.683	0.565	0.893	0.924	$\mathcal{F}_4 \succ \mathcal{F}_3 \succ \mathcal{F}_1 \succ \mathcal{F}_2$

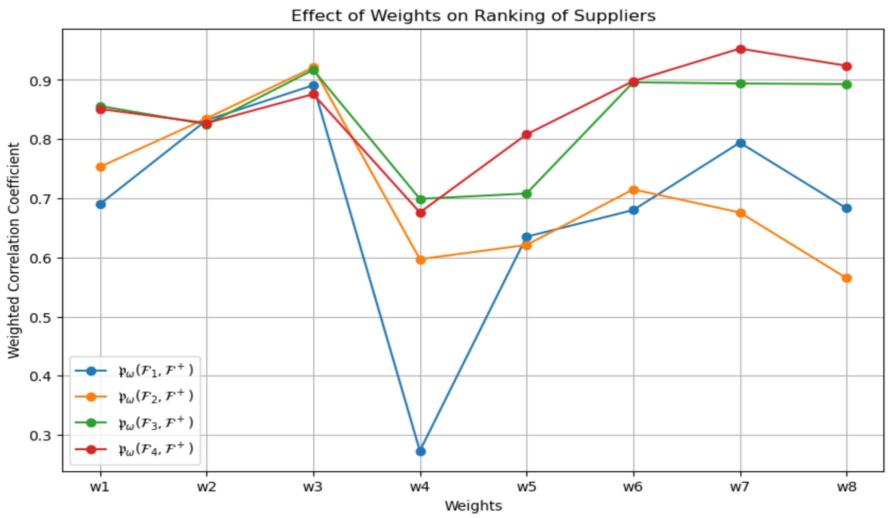

Fig 3. The line chart of the suppliers with different weights.

Based on the rankings of the four alternatives presented in *Table 3* and *Fig. 3*, it is clear that the rankings differ significantly. This variation underlines that assigning different attribute weights can lead to diverse ranking results. The influence of weights on the ranking process is thus evident, highlighting the importance of carefully considering weight allocation. DMs can modify the weights to align with their decision-making scenarios' specific priorities and requirements. By doing so, they can ensure that the ranking outcomes best reflect their strategic goals and preferences, ultimately leading to more informed and effective decision-making.

5.1.2 | Assessment of risk preference effect on ranking with constant weights

Next, we conduct a detailed analysis of how different risk preference coefficients affect the rankings. Let $\omega = (0.35,0.25,0.20,0.15,0.05)'$, the results of the ranking with different values of risk function are given in *Table 4*.

Table 4. Effect of risk preference on ranking of suppliers at constant weight.

Risk Preference (ζ)	$\mathfrak{p}_\omega(\mathcal{F}_1,\mathcal{F}^+)$	$\mathfrak{p}_\omega(\mathcal{F}_2,\mathcal{F}^+)$	$\mathfrak{p}_\omega(\mathcal{F}_3,\mathcal{F}^+)$	$\mathfrak{p}_\omega(\mathcal{F}_4,\mathcal{F}^+)$	Ranking
0	0.6737	0.8551	0.8236	0.8159	$\mathcal{F}_2 \succ \mathcal{F}_3 \succ \mathcal{F}_4 \succ \mathcal{F}_1$
0.3	0.6914	0.8378	0.8214	0.8230	$\mathcal{F}_2 \succ \mathcal{F}_4 \succ \mathcal{F}_3 \succ \mathcal{F}_1$
0.6	0.7050	0.8245	0.8197	0.8284	$\mathcal{F}_4 \succ \mathcal{F}_2 \succ \mathcal{F}_3 \succ \mathcal{F}_1$
0.9	0.7158	0.8140	0.8184	0.8327	$\mathcal{F}_4 \succ \mathcal{F}_3 \succ \mathcal{F}_2 \succ \mathcal{F}_1$
1	0.7189	0.8109	0.8180	0.8340	$\mathcal{F}_4 \succ \mathcal{F}_3 \succ \mathcal{F}_2 \succ \mathcal{F}_1$
1.2	0.7246	0.8054	0.8174	0.8363	$\mathcal{F}_4 \succ \mathcal{F}_3 \succ \mathcal{F}_2 \succ \mathcal{F}_1$
1.4	0.7296	0.8005	0.8167	0.8382	$\mathcal{F}_4 \succ \mathcal{F}_3 \succ \mathcal{F}_2 \succ \mathcal{F}_1$
1.6	0.7340	0.7962	0.8162	0.8400	$\mathcal{F}_4 \succ \mathcal{F}_3 \succ \mathcal{F}_2 \succ \mathcal{F}_1$
1.9	0.7398	0.7905	0.8155	0.8423	$\mathcal{F}_4 \succ \mathcal{F}_3 \succ \mathcal{F}_2 \succ \mathcal{F}_1$
2	0.7416	0.7889	0.8153	0.8430	$\mathcal{F}_4 \succ \mathcal{F}_3 \succ \mathcal{F}_2 \succ \mathcal{F}_1$

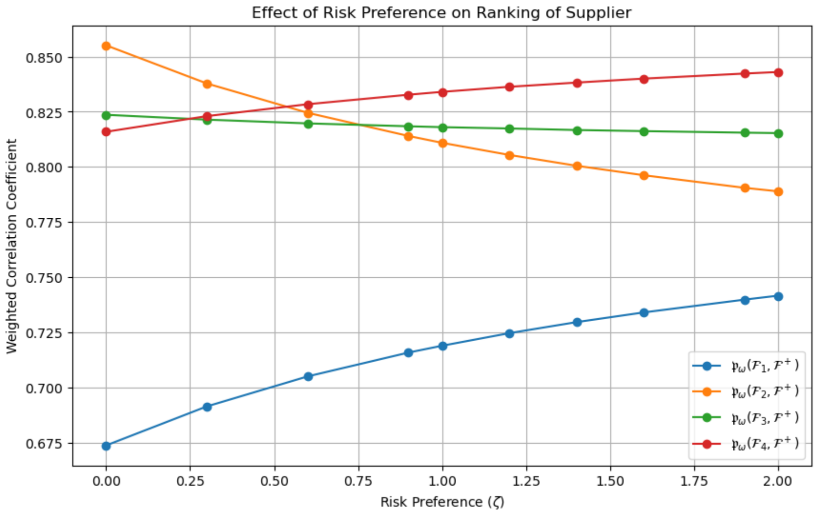

Fig. 4. The line chart of the suppliers with different risk preferences at constant weight.

The rankings of the four alternatives in *Fig. 4* show significant variation, demonstrating that the rankings change with different values of the risk preference coefficients. This variability underscores the importance of carefully considering risk preferences in decision-making. *Table 4* reveals that the risk preference coefficient ζ significantly impacts the rankings of the four alternatives. As ζ increases, indicating a more risk-averse approach, $\mathcal{F}_4$ becomes more favorable, while $\mathcal{F}_1$ becomes less attractive. Conversely, as ζ decreases, indicating a more risk-seeking approach, $\mathcal{F}_2$ maintains its relative attractiveness. By adjusting the risk preference coefficients, DMs can observe how the rankings shift, which provides valuable insights into how sensitive the outcomes are to changes in risk attitudes. This flexibility allows DMs to modify their decision-making strategies to align with their specific risk tolerance levels and objectives. The approach enhances the robustness of the decision-making process and ensures that the selected alternative aligns closely with the organization's or individual's strategic goals and risk tolerance.

6 | Conclusion

PSVNHFS is an advanced and generalized form of fuzzy set theory designed to handle the complexities of uncertainty, hesitation, and probabilistic elements in decision-making processes. Also, PSVNHFS extends traditional FS, IFS, HFS, FMS, DHFS, and SVNS by incorporating three key components, i.e., truth-hesitancy

membership value, indeterminacy-hesitancy membership value, and falsity-hesitancy membership value. Each component is associated with a probability, providing a more refined and comprehensive way to represent uncertainty and hesitation. In this paper, the CC and WCC measures for PSVNHFS are proposed. These measures handle the inherent uncertainty and hesitancy in decision-making scenarios and accommodate varying degrees of importance for different attributes by assigning specific weights. The practical application of the proposed measure is shown in the field of SCM.

The proposed measure efficiently ranks the suppliers and identifies the optimal choice by considering multiple attributes such as quality, cost, delivery time, reliability, and environmental impact. This application also highlights the flexibility of the approach, showing how adjusting the weights and risk preference coefficients can influence the final rankings and help DMs align their choices with specific strategic goals and risk tolerance levels. Exploring the application of this method in different domains, such as healthcare, finance, and environmental management, could reveal its versatility and effectiveness across various fields.

In the future, the proposed measure can be used in dealing with other MADM problems like pattern recognition, data mining, quality control & process optimization, Healthcare, and Risk management.

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Author Contributions

Rahul Thakur: Conceptualization, Data curation, Formal analysis, Methodology, Writing - original draft. Masum Raj: Validation, Investigation; writing – review & editing. S.C. Malik: Supervision; Validation; writing – review & editing; All authors have read and agreed to the published version of the manuscript.

Data Availability

All data supporting the findings of this study are included in the manuscript.

Conflicts of Interest

The authors declare that they have no conflict of interest.

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